

NON-MINIMAL EINSTEIN-YANG-MILLS-DILATON THEORY

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Abstract

We establish a new non-minimal Einstein-Yang-Mills-dilaton model, for which the Lagrangian is linear in the curvature and contains eight arbitrary functions of the scalar (dilaton) field. The self-consistent system of equations for the non-minimally coupled gauge, scalar and gravitational fields is derived. As an example of an application we discuss the model with pp-wave symmetry. Two exact explicit regular solutions of the whole system of master equations, belonging to the class of pp-wave solutions, are presented.

1 Introduction

The Einstein-Maxwell-dilaton theory and its non-Abelian generalization, Einstein-Yang-Mills-dilaton (EYMd) theory, attract serious attention, since they have a supplementary, dilatonic, degree of freedom for the structure modeling of the gravitationally coupled systems. A lot of impressive results are obtained in the framework of the *minimal* EYMd theory in the cosmological and string contexts, as well as in the application to the colored static spherically and axially symmetric objects (see, e.g., [1] - [5] and references therein). A new impetus to the development of the EYMd theory has been given by the discovery of the accelerated expansion of the Universe. Numerous attempts have been made to consider modified theories of gravitational interaction, such as $F(R)$ -gravity, Gauss-Bonnet-gravity, etc., as alternatives to the dark energy (see, e.g., [6] and references therein). One of the directions in such a generalization of the Einstein theory of gravity is connected with a *non-minimal* extension of the field theory. Some historical details, review and references, related to the non-minimal interaction of gravity with scalar and electromagnetic fields, can be found, e.g., in [7]. As for the non-minimal generalization of the Einstein-Yang-Mills theory, there are two different approaches. The first one is based on the dimensional reduction of the Gauss-Bonnet action [8], the alternative way is connected with the non-Abelian generalization of the non-minimal Einstein-Maxwell theory [9, 10] along the lines proposed by Drummond and Hathrell for the linear electrodynamics [11]. Recently in [12] one of the variants of the non-linear non-minimal generalizations of the Einstein-Yang-Mills theory has been developed, which combines, in fact, two ideas: the idea of $F(R)$ -gravity, on the one hand, and the idea of Ricci-dilaton, on the other hand. The paper [12] has stimulated in some respects the preparation of this note.

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2 Fundamentals of the non-minimal EYMd model

2.1 General concepts and definitions

We consider a non-minimal Einstein-Yang-Mills-dilaton model quadratic in the Yang-Mills field strength $F_{ik}^{(a)}$, quadratic in the derivative of the scalar (dilaton) field $\nabla_k \Phi$, and linear in the curvature. Such a model can be described self-consistently using the action functional of the type

$$S_{(\text{EYMd})} = \int d^4x \sqrt{-g} \left\{ \frac{R+2\Lambda}{\kappa} + \frac{1}{2} C_{(a)(b)}^{ikmn}(\Phi) F_{ik}^{(a)} F_{mn}^{(b)} - \mathcal{C}^{ik}(\Phi) \nabla_i \Phi \nabla_k \Phi + \mathcal{F}(\Phi) R + V(\Phi) \right\}, \quad (1)$$

which is, on the one hand, a dilatonic extension of the non-minimal Einstein-Yang-Mills model [9], and, on the other hand, a reduction of the Einstein-Yang-Mills-Higgs functional [10] to the case, when the $SU(N)$ Higgs multiplet $\Phi^{(a)}$ degenerates into the scalar singlet Φ .

The used definitions are standard: $g = \det(g_{ik})$ is the determinant of a metric tensor g_{ik} , R is the Ricci scalar, Λ is the cosmological constant, the multiplet of real tensor fields $F_{ik}^{(a)}$ describes the strength of gauge field, the symbol Φ denotes the real scalar field, associated with a dilaton, ∇_k denotes the covariant derivative, $V(\Phi)$ is a potential of the scalar field. Latin indices without parentheses run from 0 to 3, (a) and (b) are the group indices, running from (1) to $(N^2 - 1)$ for the model with $SU(N)$ symmetry. The quantities $C_{(a)(b)}^{ikmn}$ and \mathcal{C}^{ik} denote the so-called constitutive tensors for the gauge and scalar fields, respectively. They contain neither Yang-Mills strength tensor $F_{ik}^{(a)}$, nor the derivative of the scalar field. Depending on the model under consideration, these tensors can be constructed using space-time metric, its first derivatives (through the covariant derivative ∇_k), second derivatives (through the Riemann tensor R_{klm}^i , Ricci tensor R_{km} and Ricci scalar R), etc. In addition, the scalar field Φ , time-like velocity four-vector U^k and space-like director(s) N^k , and their covariant derivatives, $\nabla_k U_l$ and $\nabla_k N_l$, can be constructive elements of the constitutive tensors.

We follow the definitions of the book [13] and consider the Yang-Mills field \mathbf{F}_{mn} taking values in the Lie algebra of the gauge group $SU(N)$ (adjoint representation):

$$\mathbf{F}_{mn} = -i\mathcal{G}\mathbf{t}_{(a)}F_{mn}^{(a)}, \quad \mathbf{A}_m = -i\mathcal{G}\mathbf{t}_{(a)}A_m^{(a)}. \quad (2)$$

The generators $\mathbf{t}_{(a)}$ are Hermitian and traceless. The symmetric tensor $G_{(a)(b)}$ defined as

$$G_{(a)(b)} \equiv 2\text{Tr } \mathbf{t}_{(a)}\mathbf{t}_{(b)}, \quad (3)$$

plays a role of a metric in the group space. The tensor $F_{mn}^{(a)}$ is connected with the potentials of the gauge field $A_i^{(a)}$ by the formulas [13, 14]

$$F_{mn}^{(a)} = \nabla_m A_n^{(a)} - \nabla_n A_m^{(a)} + \mathcal{G}f_{(b)(c)}^{(a)} A_m^{(b)} A_n^{(c)}. \quad (4)$$

The symbols $f_{(b)(c)}^{(a)}$ denote the real structure constants of the gauge group $SU(N)$. The tensor $F_{(a)}^{ik}$ satisfies the relation

$$\hat{D}_k^* F_{(a)}^{ik} \equiv \nabla_k^* F_{(a)}^{ik} - \mathcal{G}f_{(b)(a)}^{(c)} A_m^{(b)} F_{(c)}^{ik} = 0. \quad (5)$$

The symbol \hat{D}_k denotes the gauge-covariant derivative. For the derivative of arbitrary tensor defined in the group space we use the following rule [15]:

$$\hat{D}_m Q_{\dots(d)}^{(a)\dots} \equiv \nabla_m Q_{\dots(d)}^{(a)\dots} + \mathcal{G}f_{(b)(c)}^{(a)} A_m^{(b)} Q_{\dots(d)}^{(c)\dots} - \mathcal{G}f_{(b)(d)}^{(c)} A_m^{(b)} Q_{\dots(c)}^{(a)\dots} + \dots. \quad (6)$$

The asterisk relates to the dual tensor

$${}^*F_{(a)}^{ik} = \frac{1}{2}\epsilon^{ikls}F_{ls(a)}, \quad (7)$$

where $\epsilon^{ikls} = \frac{1}{\sqrt{-g}}E^{ikls}$ is the Levi-Civita tensor, E^{ikls} is the completely antisymmetric symbol with $E^{0123} = -E_{0123} = 1$.

2.2 Non-minimal EYMd model based on eight arbitrary functions of the dilaton field

In the papers [9, 10] we focused on the three-, five-, six- and seven-*parameter* models, considering the parameters q_1, q_2 , etc., as phenomenological non-minimal coupling constants. Now, following the main idea of dilatonic extension of the Einstein-Maxwell and Einstein-Yang-Mills theories (see, e.g., [16]), we assume that the (non-minimal) constitutive tensors $C_{(a)(b)}^{ikmn}$ and \mathcal{C}^{ik} contain arbitrary functions of the dilaton field Φ instead of coupling constants. Our ansatz for the action functional of the non-minimal EYMd model is the following

$$S_{(\text{NMEYMd})} = \int d^4x \sqrt{-g} \left\{ \frac{R + 2\Lambda}{\kappa} + \frac{1}{2}f_0(\Phi)F_{ik}^{(a)}F_{(a)}^{ik} - \mathcal{F}_0(\Phi)\nabla_m\Phi\nabla^m\Phi + V(\Phi) + \right. \\ \left. + \mathcal{F}(\Phi)R + \frac{1}{2}\mathcal{R}^{ikmn}(\Phi)F_{ik}^{(a)}F_{mn(a)} - \mathfrak{R}^{mn}(\Phi)\nabla_m\Phi\nabla_n\Phi \right\}. \quad (8)$$

The function $f_0(\Phi)$ describes a multiplier of the dilaton-type, which is traditional for the *minimal* theory; it is placed in front of the first invariant $F_{ik}^{(a)}F_{(a)}^{ik}$ of the gauge field and is considered to be positive. In analogy with $f_0(\Phi)$ we introduce the multiplier $\mathcal{F}_0(\Phi)$ in front of the invariant $\nabla_m\Phi\nabla^m\Phi$. The function $\mathcal{F}(\Phi)$ is typical for the non-minimal extension of the scalar field theory; traditionally, one uses the term $\mathcal{F}(\Phi)R$ in the well-known form $\xi R\Phi^2$. The so-called susceptibility tensor \mathcal{R}^{ikmn}

$$\mathcal{R}^{ikmn}(\Phi) \equiv \frac{1}{2}f_1(\Phi)R(g^{im}g^{kn} - g^{in}g^{km}) + f_3(\Phi)R^{ikmn} + \\ + \frac{1}{2}f_2(\Phi)(R^{im}g^{kn} - R^{in}g^{km} + R^{kn}g^{im} - R^{km}g^{in}), \quad (9)$$

contains three arbitrary functions $f_1(\Phi), f_2(\Phi), f_3(\Phi)$ instead of curvature coupling parameters q_1, q_2, q_3 (compare with [9, 10, 17]). The tensor \mathfrak{R}^{mn}

$$\mathfrak{R}^{mn}(\Phi) \equiv f_4(\Phi)Rg^{mn} + f_5(\Phi)R^{mn}, \quad (10)$$

describes the so-called derivative coupling of the scalar field to the curvature (see, e.g., [18]), but now we consider arbitrary functions $f_4(\Phi)$ and $f_5(\Phi)$ instead of coupling parameters q_4 and q_5 (see [10]). Finally, it is worth noting, that the constitutive tensors, introduced in (1), have now the following form

$$C_{(a)(b)}^{ikmn}(\Phi) = \left[\frac{1}{2}f_0(\Phi)(g^{im}g^{kn} - g^{in}g^{km}) + \mathcal{R}^{ikmn}(\Phi) \right] G_{(a)(b)}, \quad \mathcal{C}^{mn}(\Phi) = [g^{mn}\mathcal{F}_0(\Phi) + \mathfrak{R}^{mn}(\Phi)]. \quad (11)$$

These quantities are linear in the curvature and are arbitrary functions of the dilaton field. They can be used for the definition of dilatonicallly modified color permittivities, as well as color and acoustic metrics along the lines described in [10].

2.2.1 Non-minimal extension of the Yang-Mills equations

The variation of the action $S_{(\text{NMEYMD})}$ with respect to the Yang-Mills potential $A_i^{(a)}$ yields

$$\hat{D}_k \mathcal{H}_{(a)}^{ik} = 0, \quad \mathcal{H}_{(a)}^{ik} = C_{(a)(b)}^{ikmn} F_{mn}^{(b)}, \quad (12)$$

where the term $\mathcal{H}_{(a)}^{ik}$ with constitutive tensor from (11) describes a non-minimal color excitation in analogy with electrodynamics of continuous media (see, e.g., [19]). In this context the quantity $\mathcal{R}^{ikmn}(\Phi)G_{(a)(b)}$ can be indicated as a color-dilatonic susceptibility tensor.

2.2.2 Non-minimal extension of the scalar field equations

The variation of the action $S_{(\text{NMEYMD})}$ with respect to the scalar Φ gives the master equation for the dilaton field

$$\begin{aligned} \nabla_m \{ [g^{mn} \mathcal{F}_0(\Phi) + \mathfrak{R}^{mn}(\Phi)] \nabla_n \Phi \} = & -\frac{1}{2} \frac{d}{d\Phi} V(\Phi) - \frac{1}{2} R \frac{d}{d\Phi} \mathcal{F}(\Phi) + \frac{1}{2} \nabla_m \Phi \nabla^m \Phi \frac{d}{d\Phi} \mathcal{F}_0(\Phi) - \\ & - \frac{1}{4} \left[F_{(a)}^{mn} F_{mn}^{(a)} \frac{d}{d\Phi} f_0(\Phi) + F_{ik(a)} F_{mn}^{(a)} \frac{d}{d\Phi} \mathcal{R}^{ikmn}(\Phi) \right] - \frac{1}{2} \nabla_m \Phi \nabla_n \Phi \frac{d}{d\Phi} \mathfrak{R}^{mn}(\Phi). \end{aligned} \quad (13)$$

Clearly, this equation is coupled to the non-minimally extended Yang-Mills equation (12), when the constitutive tensor (11) is dilatonicly extended.

2.2.3 Master equations for the gravitational field

The variation of the action $S_{(\text{NMEYMD})}$ with respect to the metric gives the non-minimally extended equations of the Einstein type

$$\begin{aligned} \left(R_{ik} - \frac{1}{2} R g_{ik} \right) \cdot [1 + \kappa \mathcal{F}(\Phi)] = & \Lambda g_{ik} + \kappa (\nabla_i \nabla_k - g_{ik} \nabla_m \nabla^m) \mathcal{F}(\Phi) + \\ & + \kappa \left[T_{ik}^{(YM)} + T_{ik}^{(\Phi)} + T_{ik}^{(\text{NonMin})} \right]. \end{aligned} \quad (14)$$

Here the term $T_{ik}^{(YM)}$

$$T_{ik}^{(YM)} \equiv f_0(\Phi) \left[\frac{1}{4} g_{ik} F_{mn}^{(a)} F_{(a)}^{mn} - F_{in}^{(a)} F_{k(a)}^n \right], \quad (15)$$

is a stress-energy tensor of Yang-Mills field extended by the dilatonic multiplier. The term $T_{ik}^{(\Phi)}$

$$T_{ik}^{(\Phi)} = \mathcal{F}_0(\Phi) \left[\nabla_i \Phi \nabla_k \Phi - \frac{1}{2} g_{ik} \nabla_m \Phi \nabla^m \Phi \right] + \frac{1}{2} V(\Phi) g_{ik}, \quad (16)$$

is a dilatonicly extended stress-energy tensor of the scalar field. The term $T_{ik}^{(\text{NonMin})}$ can be decomposed into five parts

$$T_{ik}^{(\text{NonMin})} = T_{ik}^{(I)} + T_{ik}^{(II)} + T_{ik}^{(III)} + T_{ik}^{(IV)} + T_{ik}^{(V)}, \quad (17)$$

which are enumerated corresponding to the functions f_1, f_2, \dots, f_5 :

$$T_{ik}^{(I)} = f_1(\Phi) R \left[\frac{1}{4} g_{ik} F_{mn}^{(a)} F_{(a)}^{mn} - F_{in}^{(a)} F_{k(a)}^n \right] - \frac{1}{2} f_1(\Phi) R_{ik} F_{mn}^{(a)} F_{(a)}^{mn} +$$

$$+ \frac{1}{2} [\hat{D}_i \hat{D}_k - g_{ik} \hat{D}^l \hat{D}_l] [f_1(\Phi) F_{mn}^{(a)} F_{(a)}^{mn}] , \quad (18)$$

$$\begin{aligned} T_{ik}^{(II)} = & -\frac{1}{2} g_{ik} \left\{ \hat{D}_m \hat{D}_l [f_2(\Phi) F^{mn(a)} F_{n(a)}^l] - f_2(\Phi) R_{lm} F^{mn(a)} F_{n(a)}^l \right\} \\ & - f_2(\Phi) F^{ln(a)} [R_{il} F_{kn(a)} + R_{kl} F_{in(a)}] - f_2(\Phi) R^{mn} F_{im}^{(a)} F_{kn(a)} - \frac{1}{2} \hat{D}^m \hat{D}_m [f_2(\Phi) F_{in}^{(a)} F_k^{n(a)}] \\ & + \frac{1}{2} \hat{D}_l \left\{ \hat{D}_i [f_2(\Phi) F_{kn}^{(a)} F_{(a)}^{ln}] + \hat{D}_k [f_2(\Phi) F_{in}^{(a)} F_{(a)}^{ln}] \right\} , \end{aligned} \quad (19)$$

$$\begin{aligned} T_{ik}^{(III)} = & \frac{1}{4} f_3(\Phi) g_{ik} R^{mnl s} F_{mn}^{(a)} F_{ls(a)} - \frac{3}{4} f_3(\Phi) F^{ls(a)} [F_i^{n(a)} R_{knls} + F_k^{n(a)} R_{inls}] \\ & - \frac{1}{2} \hat{D}_m \hat{D}_n \left\{ f_3(\Phi) [F_i^{n(a)} F_k^{m(a)} + F_k^{n(a)} F_i^{m(a)}] \right\} , \end{aligned} \quad (20)$$

$$\begin{aligned} T_{ik}^{(IV)} = & f_4(\Phi) \left[\left(R_{ik} - \frac{1}{2} R g_{ik} \right) \nabla_m \Phi \nabla^m \Phi + R \nabla_i \Phi \nabla_k \Phi \right] \\ & + (g_{ik} \nabla_n \nabla^n - \nabla_i \nabla_k) [f_4(\Phi) \nabla_m \Phi \nabla^m \Phi] , \end{aligned} \quad (21)$$

$$\begin{aligned} T_{ik}^{(V)} = & f_5(\Phi) \nabla_m \Phi [R_i^m \nabla_k \Phi + R_k^m \nabla_i \Phi] + \\ & + \frac{1}{2} g_{ik} [\nabla_m \nabla_n - R_{mn}] [f_5(\Phi) \nabla^m \Phi \nabla^n \Phi] \\ & - \frac{1}{2} \nabla^m \left\{ \nabla_i [f_5(\Phi) \nabla_m \Phi \nabla_k \Phi] + \nabla_k [f_5(\Phi) \nabla_m \Phi \nabla_i \Phi] - \nabla_m [f_5(\Phi) \nabla_i \Phi \nabla_k \Phi] \right\} . \end{aligned} \quad (22)$$

Straightforward calculations, based on the Bianchi identities and on the properties of the Riemann tensor, show that the equality

$$\nabla^k \left\{ \frac{(\nabla_i \nabla_k - g_{ik} \nabla_m \nabla^m) \mathcal{F}(\Phi) + T_{ik}^{(YM)} + T_{ik}^{(\Phi)} + T_{ik}^{(\text{NonMin})}}{1 + \kappa \mathcal{F}(\Phi)} \right\} = 0 \quad (23)$$

is satisfied identically, when $F_{ik}^{(a)}$ is a solution of the Yang-Mills equations (12), and Φ is the solution of (13). Thus, we deal with self-consistent system of master equations (12), (13) and (14)- (22), which form the non-minimally extended Einstein-Yang-Mills-dilaton model with eight arbitrary functions $\mathcal{F}_0(\Phi)$, $\mathcal{F}(\Phi)$, $f_0(\Phi)$, $f_1(\Phi)$, ... $f_5(\Phi)$.

3 Application of the non-minimal EYMd theory to the model with pp-wave symmetry

3.1 Reduction of master equations

Let us consider now a plane-symmetric space-time associated usually with a gravitational radiation. We assume the metric to be of the form

$$ds^2 = 2dudv - L^2(u) [e^{2\beta(u)} (dx^2)^2 + e^{-2\beta(u)} (dx^3)^2] , \quad (24)$$

where $u = (t - x^1)/\sqrt{2}$ and $v = (t + x^1)/\sqrt{2}$ are the retarded and advanced time, respectively. This space-time is known to admit the G_5 group of isometries [20], and three Killing four-vectors,

ξ^k , $\xi_{(2)}^k$ and $\xi_{(3)}^k$ form three-dimensional Abelian subgroup G_3 . The four-vector ξ^k is the null one and covariantly constant, i.e.,

$$\xi^k = \delta_v^k, \quad g_{kl} \xi^k \xi^l = 0, \quad \nabla_l \xi^k = 0. \quad (25)$$

The four-vectors $\xi_{(\alpha)}^k$ ($\alpha = 2, 3$) are space-like and orthogonal to ξ^k and to each other, i.e.,

$$\xi_{(\alpha)}^k = \delta_\alpha^k, \quad g_{kl} \xi_{(2)}^k \xi_{(3)}^l = 0, \quad g_{kl} \xi^k \xi_{(\alpha)}^l = 0. \quad (26)$$

The non-vanishing components of the Ricci and Riemann tensors are, respectively

$$R_{uu} = R_{u2u}^2 + R_{u3u}^3, \quad R_{u2u}^2 = - \left[\frac{L''}{L} + (\beta')^2 \right] - \left[2\beta' \frac{L'}{L} + \beta'' \right], \quad (27)$$

$$R_{u3u}^3 = - \left[\frac{L''}{L} + (\beta')^2 \right] + \left[2\beta' \frac{L'}{L} + \beta'' \right], \quad R = 0. \quad (28)$$

We consider a *toy-model*, which satisfies the following requirements. *First*, the potentials of the Yang-Mills field are parallel in the group space [21], i.e.,

$$A_k^{(a)} = q^{(a)} A_k, \quad G_{(a)(b)} q^{(a)} q^{(b)} = 1, \quad q^{(a)} = \text{const}. \quad (29)$$

Second, the vector field A_k and scalar field Φ inherit the symmetry of the space-time, i.e., the Lie derivatives of these quantities along generators of the group G_3 , $\{\xi\} \equiv \{\xi^k, \xi_{(2)}^k, \xi_{(3)}^k\}$, are equal to zero:

$$\mathcal{L}_{\{\xi\}} A_k = 0, \quad \mathcal{L}_{\{\xi\}} \Phi = 0. \quad (30)$$

Third, the vector field A^k is transverse, i.e.,

$$A^k = - \left[A_2 \xi_{(2)}^k + A_3 \xi_{(3)}^k \right], \quad \xi^k A_k = 0. \quad (31)$$

Fourth, the potential of the scalar field is equal to zero, $V(\Phi) = 0$. *Fifth*, the cosmological constant is absent, $\Lambda = 0$. These five requirements lead to the following simplifications.

(i) The fields A_k and Φ depend on the retarded time u only; there are two non-vanishing components of the field strength tensor

$$F^{(a)ik} = q^{(a)} \left[\left(\xi^i \xi_{(2)}^k - \xi^k \xi_{(2)}^i \right) A_2'(u) + \left(\xi^i \xi_{(3)}^k - \xi^k \xi_{(3)}^i \right) A_3'(u) \right], \quad (32)$$

the invariant $F_{ik}^{(a)} F_{(a)}^{ik}$ as well as the terms $\mathcal{R}^{ikmn} F_{mn}^{(a)}$ are equal to zero.

(ii) The equations (12) and (13) are satisfied identically.

(iii) The non-minimal terms $T_{ik}^{(I)}, \dots, T_{ik}^{(V)}$ (18)-(22) disappear, and the functions $f_1(\Phi), f_2(\Phi), \dots, f_5(\Phi)$, being non-vanishing, happen to be hidden, i.e., they do not enter the equations for the gravity field.

After such simplifications the non-minimal equations for the gravity field (14)-(22) reduce to one equation

$$\frac{L''}{L} + (\beta')^2 = -\frac{1}{2} \kappa T(u), \quad (33)$$

where

$$T(u) = \left\{ \frac{f_0(\Phi)}{L^2} \left[e^{-2\beta} (A_2')^2 + e^{2\beta} (A_3')^2 \right] + (\Phi')^2 \left[\mathcal{F}_0(\Phi) + \frac{d^2}{d\Phi^2} \mathcal{F}(\Phi) \right] + \Phi'' \frac{d}{d\Phi} \mathcal{F}(\Phi) \right\} [1 + \kappa \mathcal{F}(\Phi)]^{-1}. \quad (34)$$

For this model $A_2(u)$, $A_3(u)$, $\Phi(u)$ are arbitrary functions of the retarded time, and the prime denotes the derivative with respect to u .

3.2 Regular solutions

Let us consider a model with $L(u) \equiv 1$. It can be indicated as a regular model, since $\det(g_{ik}) = -L^4 \equiv -1$ and can not vanish, contrary to the standard situation with gravitational pp-waves [22]. When $\mathcal{F} = 0$, and $\mathcal{F}_0(\Phi)$, $f_0(\Phi)$ are positive functions, there are no real solutions of the equation (33) with $L = 1$, since the right-hand side is negative for arbitrary moment of the retarded time (see (34)). Nevertheless, such a possibility appears in the non-minimal case. Below we consider two explicit examples of the exact regular solutions.

3.2.1 First explicit example

Let the scalar field take a constant value $\Phi_0 \neq 0$, then the functions

$$A_2(u) = A_2(0) e^{\beta(u)}, \quad A_3(u) = A_3(0) e^{-\beta(u)}, \quad (35)$$

give the exact solution of (33) for arbitrary $\beta'(u)$, when

$$-\kappa \mathcal{F}(\Phi_0) = 1 + \frac{\kappa}{2} f_0(\Phi_0) [A_2^2(0) + A_3^2(0)]. \quad (36)$$

For a given negative function $\mathcal{F}(\Phi)$ and positive function $f_0(\Phi)$, this equality predetermines some special value of the dilaton field, Φ_0 . The function $\beta(u)$ is now arbitrary, and we can use, for instance, the periodic finite function

$$\beta(u) = \frac{1}{2} h (1 - \cos 2\lambda u), \quad \beta(0) = 0, \quad \beta'(0) = 0. \quad (37)$$

The metric for this non-minimal model is periodic and regular

$$ds^2 = 2dudv - [e^{2h \sin^2 \lambda u} (dx^2)^2 + e^{-2h \sin^2 \lambda u} (dx^3)^2], \quad (38)$$

the potentials of the Yang-Mills field (35) are also periodic and regular.

3.2.2 Second explicit example

Let the dilaton field be periodic, i.e.,

$$\Phi(u) = \Phi_0 \sin \lambda u, \quad (39)$$

and the color wave be circularly polarized, i.e.,

$$A'_2(u) = E_0 e^{\beta(u)} \cos \omega u, \quad A'_3(u) = E_0 e^{-\beta(u)} \sin \omega u. \quad (40)$$

The wave is called circularly polarized in analogy with electrodynamics, since the function

$$E^2(u) \equiv -g^{ik} A'_i A'_k = e^{-2\beta} (A'_2)^2 + e^{2\beta} (A'_3)^2 = E_0^2 \quad (41)$$

is constant for such a field. Let us consider the functions $\mathcal{F}_0(\Phi)$, $f_0(\Phi)$ and $\mathcal{F}(\Phi)$ to be of the form

$$\mathcal{F}_0(\Phi) = 1, \quad f_0(\Phi) = 1 + \left(\frac{\Phi(u)}{\Phi_0} \right)^2, \quad \mathcal{F}(\Phi) = - \left(1 + \frac{2}{\kappa \Phi_0^2} \right) \Phi^2(u), \quad (42)$$

and assume for simplicity, that $E_0 = \Phi_0 \lambda$. Then the equations (33) and (34) yield

$$(\beta'(u))^2 = 2\lambda^2, \quad \beta(u) = \pm\sqrt{2}\lambda u. \quad (43)$$

Thus, the metric takes the form

$$ds^2 = 2dudv - \left[e^{\pm 2\sqrt{2}\lambda u} (dx^2)^2 + e^{\mp 2\sqrt{2}\lambda u} (dx^3)^2 \right], \quad (44)$$

and the space-time is symmetric [20], i.e., all the non-vanishing components of the Riemann tensor are constant

$$R^2_{u2u} = R^3_{u3u} = -(\beta')^2 = -2\lambda^2 = \frac{1}{2}R_{uu}. \quad (45)$$

The Yang-Mills potentials are quasi-periodic functions

$$A_2(u) = \frac{E}{(\omega^2 + 2\lambda^2)} \left[e^{\pm\sqrt{2}\lambda u} (\sqrt{2}\lambda \cos \omega u + \omega \sin \omega u) - \sqrt{2}\lambda \right], \quad A_3(0) = 0, \quad (46)$$

$$A_3(u) = \frac{E}{(\omega^2 + 2\lambda^2)} \left[e^{\mp\sqrt{2}\lambda u} (\sqrt{2}\lambda \sin \omega u - \omega \cos \omega u) + \omega \right], \quad A_3(0) = 0. \quad (47)$$

This solution is also free of singularity at the finite moments of the retarded time.

4 Conclusions

The Lagrangian of the presented non-minimal Einstein-Yang-Mills-dilaton model includes eight arbitrary functions depending on the dilaton field, thus, we deal with a wide freedom of modeling. The first (simplest) example of the application shows, that for the model with pp-wave symmetry five of the eight arbitrary functions happen to be hidden, i.e., they do not enter the master equations. Nevertheless, the presence of three remained functions of the dilatonic field allows us to find exact explicit solutions of the whole self-consistent system of master equations, which can be indicated as regular (quasi)periodic solutions of the pp-wave type. This means, that the dilatonic extension of the non-minimal field theory seems to be a promising instrument for its modification.

We assume, that this non-minimal EYMd model might be fruitful for cosmological applications, especially, for the explanation of the accelerated expansion of the Universe and dark energy phenomenon. We also believe, that the mentioned wide freedom of modeling in the framework of the non-minimal EYMd theory can be used in searching for new regular solutions, describing colored static spherically symmetric objects. We intend to discuss these problems in detail in further papers.

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